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An almost physical interpretation of the integrand of the n-point Veneziano model.

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Abstract: It is shown that the integrand $\prod (u_{ik})^{-\alpha_{ik}}$ of the integral representing the n-point Veneziano model is equal to the exponential of the extremal value of a certain integral. In fact the form of the exponent is that of the rate of heat generation under certain conditions in a metallic disk to be indentified with the unit circle in our manifestly cyclically symmetric parametrisation of the Veneziano n-point function. Heuristically a physical interpretation of this form is given in a model in which hadrons are concieved of as one dimensional structures. An approximation to a class of infinitely complicated Feynman diagrams can also be used. A step towards an understanding of the meaning of the Møbius invariance of the integrand of the manifestly cyclically symmetric representation is made.

Introduction & A representation of the Veneziano model

The main mathematical observation of this note is that the integrand $\prod_{\substack{all \\ hooks(i,k)}} (u_{ik})^{-\alpha_{ik}}$ of the integral representations of n-point generalized Veneziano - Bardakçi - Ruegg - Virasoro model [1)-3)] invented by Chan and Tsou [4)] and by several other groups [5)-8)] independently can be written in an almost physically interpretable manner. The transscription is stated and proven in this section. In section II it is used to interprete the Veneziano model as a relativistic model for mesons that are in reality infinitely long chain molecules. In section III we argue that if the amplitude shall be approximated by Feynman diagrams of some (simple) large scale structure and the limit of infinetely complex diagrams is to be taken, the only non-trivial possibility is the one implying that hadrons are threads or chain molecules.

Our considerations are closest connected with the manifestly cyclically symmetric representation [9)-10)] of the generalized Veneziano model. In this formulation the scattering amplitude for n scalar mesons is written as a linear combination of terms (one for each quark-diagram) of the form:

1) $$B^{(n)} = \int_{\theta_1 \leq \theta_2 \leq \ldots \leq \theta_n \leq \theta_1 + 2\pi} \prod_{\substack{all \\ hooks(i,k)}} (u_{ik})^{-\alpha_{ik}-1} \, dF_n \Big/ dF_3$$

$$= \int_{\theta_1 \leq \ldots \leq \theta_n \leq \theta_1 + 2\pi} \prod_{all\, hooks(i,k)} (u_{ik})^{-\alpha_{ik}} \, d\hat{F}_n \Big/ dF_3$$

where

2) $$dF_n = \frac{\prod_{J=1}^{n} dz_J}{\prod_{J=1}^{n} (z_{J+2} - z_J)} \quad \text{and} \quad d\hat{F}_n = \frac{-\prod_{J=1}^{n} dz_J}{\prod_{J=1}^{n} (z_{J+1} - z_J)}$$

(indices calculated modulo n) and the n variables $z_J = e^{i\theta_J}$ one corresponding to each external meson, runs on the unit circle in the complex plane. The symbol dF_3 just has the purpose of removing an infinite constant factor from the integrals which would otherwise diverge. The Regge trajectories are supposed linear with the universal slope α', that is

$$\alpha_{ik} = \alpha'(s_{ik} - \mu_{ik}^2)$$

The indices (i,k) denote the quark and antiquark which are present in the channel of α_{ik}.

The " conjugate" variables [4] u_{ik} are of the form

3) $$u_{ik} = \frac{Z_I - Z_{\bar{K}}}{Z_I - Z_K} : \frac{Z_{\bar{I}} - Z_{\bar{K}}}{Z_{\bar{I}} - Z_K}$$

where the suffixes denotes the external mesons, I is the meson containing the quark i, $\bar{I}$ is the one containing the antiparticle of quark i. Analogously for K and $\bar{K}$. (see fig 1)

Now we take the disk surrounded by the unit circle in the complex plane as a two dimensional space that allows flows of the additively conserved quantum numbers to go through it. For instance we shall talk about flux densities of fourmomentum describing a stationary flow of four types of fluid inside the unit circle, the flux density in the direction of the real axis is called $f_1^{\mu}(\underline{x})$, that in the direction of the imaginary one $f_2^{\mu}(\underline{x})$. Together these 2 times 4 fields are written as the vector $\vec{f}^{\mu}(\underline{x})$. As coordinates are used the couple of real variables $\underline{x}=(x_1,x_2)$ defined by the relation

4) $$z=x_1+ix_2.$$

The fluids should be so that the continuity equation

5) $$\operatorname{div} \vec{f}^{\mu}(\underline{x}) = \frac{\partial}{\partial x_1} f_1^{\mu}(\underline{x}) + \frac{\partial}{\partial x_2} f_2^{\mu}(\underline{x}) = 0$$

holds, and at the boundary

(6) $$\vec{n} \cdot \vec{f}^{\mu} = 0$$

where $\vec{n}$ is the normal to the circle periphery, except at the points z_J $(J=1,2,\ldots,N)$ corresponding to the external mesons where a flux of fourmomentum equal to the fourmomentum of meson number J flows into the disk. Analogously $\vec{g}^{\Sigma}$ sehould describe a flow of some charge Q_s.

Under the constraints of these boundary conditions and the continuity equation (5) we vary the fields $\vec{f}^{\mu}$ and $\vec{g}^{\Sigma}$ so as to make the integral

(3)(7) $$\int_{\bigcirc} \mathcal{L}\, d\underline{x}$$

exstremal, that is to make

4)(8) $$\delta \int_{\bigcirc} \mathcal{L}\, d\underline{x} = 0$$

Here

$$\mathcal{L} = +\pi\alpha' \, \vec{f}^{\mu}\cdot\vec{f}_{\mu} + \pi\alpha' \sum_{s,r} d_{rs} \, \vec{g}^{r}\cdot\vec{g}^{s} \tag{9}$$

Here α' and d_{rs} are constants, the constant α' is to be identified with a universal slope of Regge trajectories and the matrix has to be arranged so that the lowest mass scalar meson with Q_s quanta of charge of type s has the mass square

$$\mu^2 = \sum_{r,s} d_{rs} \, Q_s \, Q_r \tag{10}$$

Now it is our result that the integrand of the n - point Veneziano model is just the exponential of the extremal value of the integral (7) i. e. We have the identity

$$\prod (u_{ik})^{-\alpha_{ik}} = \exp\left(\text{extr. val. of} \int_{\circ} \mathcal{L}\, d\underline{x} \right) \tag{11}$$

Dr. D. B. Fairlie has pointed out to us that the extremized integral is a linear combination of terms that are just of the form of certain rates of heat generation in the disk identified with the unit circle, when electric currents are led in from electrodes at the positions Z_J $(J=1,2,\dots,N)$. In fact if we e. g. arrange that the current flowing in through the J'th electrode (at position Z_J) equals the component of the momentum of the J'th external meson in the 3-direction, then the rate of heat generation in the homogeneous metallic disk is proportional the extremal value of the integral

(12) $$\int_{\mathfrak{S}} \vec{f}^{3} \cdot \vec{f}^{3} \, dx$$

Similarly electric currents corresponding to the other momenta and to the eigenvalues of $(d_{\gamma d})$ are needed to make the whole extremized integral a linear combination of heat generation rates. The flows of currents given by the law of Ohm are exactly those corresponding to the $\vec{f}^{\mu}$ and $\vec{g}^{\lambda}$ that give the extremal value of the integral (7).

It is of interest to notice that an integral

(13) $$\int_{\mathcal{S}} \mathcal{L} \, dx$$

where $\mathcal{L}$ is given by equation (9), but where $\mathcal{S}$ can be any Riemann surface, is conformal invariant in the following sense:

Let there exist a conformal mapping+) ν of the surface piece $\mathcal{S}$ onto another one $\mathcal{S}'$. To a piece of curve σ in $\mathcal{S}$ there corresponds then a piece of curve σ' in $\mathcal{S}'$ under the conformal mapping ν. Corresponding to the fields $\vec{f}^{\mu}, \vec{g}^{\lambda}$ defined over $\mathcal{S}$ and obeying continuity equations like (5) we define a set of fields with the same names on the surface $\mathcal{S}'$ satisfying the condition

+) By a conformal mapping we understand a one-to-one and onto mapping locally preserving the angle between two curves together with the sense of rotation of a tangent.

that the total flow of each kind across any σ shall be equal to that across the corresponding σ'.

This condition is easily seen to determine the fields $\vec{f}^{\mu}$ and $\vec{g}^{1}$ on the surface $\mathcal{S}'$ completely from those on $\mathcal{S}$.

The conformal invariance means that then

(14) $$\int_{\mathcal{S}} \mathcal{L}\, d\underline{x} = \int_{\mathcal{S}'} \mathcal{L}\, d\underline{x}'$$

According to this conformal invariance one can obtain the expression $\prod (u_{ik})^{-\alpha_{ik}}$ from an extremized integral

(15) $$\text{ext val of} \int_{\mathcal{S}} \mathcal{L}\, d\underline{x},$$

not only as we have seen above when $\mathcal{S}$ is just the unit disk, but from any simply connected domain $\mathcal{S}$.

Fig. 2 illustrates a possible domain $\mathcal{S}$.

Also the conformal invariance immediately implies that the expression (11) must be invariant under conformal mapping of the unit circle onto another circle or onto itself, but this accurately means invariance under Møbius transformations. This invarians is of course obvious for the left hand side $\prod (u_{ik})^{-\alpha_{ik}}$, since it is only a function of anharmonic ratios.

An elegant proof (pointed out by D. B. Fairlie) of the mathematical relation (11) goes by choosing for the region $\mathcal{G}$ the upper half plane $\{z / \mathcal{I}m\, z > 0\}$ and using the transscription that is essentially contained in references 9 and 10, namely

(16)
$$\prod (u_{ik})^{-\alpha_{ik}} = \prod_{A,B} (z_A - z_B)^{-\alpha' p_A \cdot p_B + \alpha' \sum_{s,r} d_{rs} Q_r(A) Q_s(B)}$$

of the left hand side of equation (11). (The z_A , z_B etc. being on the boundary of the conducting sheet are now real). Remark that if the external mesons are just the lightest ones and onshell, the exponents

(17)
$$-\alpha' p_A^2 + \alpha' \sum_{r,s} d_{rs} Q_s(A) Q_r(A)$$

of the root $z_A - z_A = 0$ just vanishes, and thus the expression (16) is welldefined. Correspondingly $\alpha_{i\,i+1} = 0$ in the onshell case and thus the factors $(u_{i\,i+1})^{-\alpha_{i+1\,i}}$ on the left hand side of equation (16) is one and can be removed.

We introduce the stream functions U^μ and U^s

by the defining equations

$$f_1^\mu = -\frac{\partial U^\mu}{\partial x_2}$$

(18)

$$f_2^\mu = \frac{\partial U^\mu}{\partial x_1}$$

$$g_1^s = -\frac{\partial U^s}{\partial x_2}$$

(19)

$$g_2^s = \frac{\partial U^s}{\partial x_1}$$

These stream functions exist because of the continuity equations[11)].

Substituting (18 - 19) into formula (5) the variational principle (4) ~~gix~~ gives the Euler equations

$$\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}\right) U = \Delta U = 0$$

(20)

for all the U's i. e. both U^μ and U^s. Along the boundaries (in the case considered the real axis) the U's must be piecewise constant, because a line of flow are bound to follow each piece of the boundaries. At $z = z_J$ for some J the U's step by amounts given by the corresponding meson J. So U is known along the boundary up to an additive constant that is without significans because only derivatives of U occur in $\mathcal{L}$.

In fact the restriction of the U_1' to the boundary can be written

$$U^{\mu}(z) = -\sum_A \theta(z_A - z)\, p_A^{\mu} \quad \text{for } z \in \dot{R} \cup \{\infty\} \tag{21}$$

$$U^{\mathfrak{s}}(z) = -\sum_A \theta(z_A - z)\, Q_{\mathfrak{s}}(A) \quad \text{for } z \in \dot{R} \cup \{\infty\} \tag{22}$$

(if the external mesons all are counted as incomming.) Since

$$\theta(z_A - z) = \mathrm{Im}\left[-\frac{1}{\pi}\log(z - z_A)\right] \quad \text{for } z \in \dot{R} \cup \{\infty\} \tag{23}$$

we have

$$U^{\mu}(z) = \mathrm{Im}\,\frac{1}{\pi}\sum_A p_A^{\mu} \log(z - z_A)$$

(24)

$$U^{\mathfrak{s}}(z) = \mathrm{Im}\,\frac{1}{\pi}\sum_A Q_{\mathfrak{s}}(A) \log(z - z_A)$$

on the boundary (the real axis), but these espressions (24) extended to all of the half plane are obviously harmonic functions and thus obey the Euler equations (19) . Since U is uniquely determined from (19) and the boundary values (this is in fact the dirichlet problem) expressions (24) give the U's corresponding to the extremal value of the integral $\int_{\mathcal{S}} \mathcal{L} d\underline{x}$.

If we want to interprete $\mathcal{L}$ of equation (9) as a heat generation rate per unit area of the surface $\mathcal{S}$, we must take $\pm\pi\alpha'$ and $\pi\alpha' d_{\nu s}$ as specific resistance and the potentials become

$$V_\mu = -\pi\alpha' \,\mathrm{Re} \sum_A p_{A\mu} \log (Z - Z_A)$$

(25) $$V^s = \pi\alpha' \,\mathrm{Re} \sum_\nu \sum_A{}' Q_\nu(A)\, d_{s\nu} \log(Z - Z_A)$$

In fact these potentials satisfy the laws of Ohm

$$-\pi\alpha' \vec{J}_\mu = -\operatorname{grad} V_\mu$$

(26) $$\pi\alpha' \sum_\nu d_{s\nu} \vec{J}^\nu = -\operatorname{grad} V^s$$

because of the Cauchy Riemann relations and relations (18 - 19).

We have proven that such potentials exist, provided the currents

correspond to the extremal value of the integral of the form (13).
Let us express the fact that the total heat generation rate equals
the effect transmitted to the sheet

(27)
$$\underset{\substack{\text{Half}\\\text{plane}}}{\text{extr. val} \int} \mathcal{L}\, d\underline{x} = \sum_A p_A^{\mu} V_{\mu}(z_A) + \sum_s \sum_A Q_s(A)\, V^s(z_A).$$

Substituting equations (25) leads to

(28)
$$\text{extr. val} \int \mathcal{L}\, d\underline{x} = -\pi\alpha' \sum_{A,B} p_A^{\mu} p_{B\mu} \log|z_A - z_B| + \pi\alpha' \sum_{r,s} \sum_{A,B} Q_r(A)\, Q_s(B)\, d_{rs} \log|z_A - z_B|$$

Putting this in as exponent immediately gives the right handside

of equation (16) and so we have proven equation (11).

Section II

Physical interpretation

Using our formula (11) it is possible to interprente the generalized Veneziano within a model in which the mesons are thread like structures (allthough we are not able to say yet which satelite terms might follow from such a model).

Hadronic interactions are conceived of then as proces - ses in which threads are connected at the end points into (at first) longer threads which are then again split up into (at first)shorter threads. In fact the mapping $V^{\mu}: \mathcal{G} \rightarrow$ "Minchowski space" des- cribed by the potentials of equation (25) could be conceived of as describing the time track of a thread moving around in physical three space, more correctly we do not necessarily have the time track of a single thread but rather a system of threads. However, we must imagine that normaly these threads are actually off- shell, so that a reaction among a set of mesons is considered the results of a tunnel effect. This is connected with the fact that the expo- nent of the right hand side of (11) is real and thus gives an exponential damping of the amplitude for a certain "motion" of the thread material.

In reality we must assume that the thread has the property of being massless, when it is unstressed and with no charge distributed along it.

The first approximation of the penetration amplitude through a tunnel can be written apart from a constant factor in the form

(29) $$\exp\left(-mi\int \Delta E(t)\,dt\right)$$

Here the integral $\int \Delta E(t)\,dt$ is to be evaluated for all motions of the mechanical system (the classical analog) through its configuration space from the (given) initial to the (given) final situation. By a motion is here just understood a mapping of the time axis into configuration space and we just require the endpoints to be mapped into the initial and final point in configuration space. The symbol $\Delta E(t)$ means the break down of energy conservation at time t. Of course for any motion leading across a barrier energy conservation is broken in classical mechanics the deviation of the classicaly calculated energy and the imposed ones is called $\Delta E(t)$. Remark that an expression like $\int \Delta E(t)\,dt$ makes no precise sense for any set of initial and final states allowed by the uncertainty principle.

Nevertheless expression (29) is an approximation to the pene~~xxxxxxxx~~tration amplitude. The intuitive content of it is that $\int \Delta E(t)\, dt$ is a measure of how seriously the law of energy conservation has been broken and that the penetration amplitude must be the smaller the more criminal the penetration in order that the crimes be so seldom that the uncertainty principle can prevent their detection.

Since barrier is passed almost exclusively near the easiest way a penetration is almost always roughly a one dimensional problem and we can essentially prove (29) by illustrating its truth for a one dimensional configuration space. It is ~~xxxxxx~~ wellknown that the penetration amplitude for a particle in a one dimensional space crossing a potential barrier is given roughly by the factor

(30) $$\exp\left(-\int\sqrt{2m|E-V|}\,dx\right)$$

(In fact this is the JWKB approximation). Here V is the potential, E the imposed energy and m the mass of the particle, x its position. For a motion x(t) we find ~~xxxxx~~ for this system

(31) $$\Delta E(t) = -E + \frac{1}{2} m \left(\frac{dx(t)}{dt}\right)^2 + V(x(t))$$

We minimise the integral

(32) $$\int \Delta E(t)\, dt = \int_a^b \Delta E(t) \left(\frac{dx(t)}{dt}\right)^{-1} dx(t)$$

by requiring that

(33) $$\frac{dx(t)}{dt} = \sqrt{\frac{2(V-E)}{m}}$$

This leads to

(34) extremal val. of $$\int \Delta E(t)\, dt = \int_a^b \Big[\frac{1}{2} m \sqrt{\frac{V(x(t)) - E}{\frac{1}{2} m}} + (V(x(t)) - E) \sqrt{\frac{m}{2(V(x(t)) - E)}} \Big] dx$$

$$= \int_a^b \sqrt{2m(V-E)}\, dx$$

but this just gives us (30).

Now the energy of a thread or stick can be written as an integral along the thread. So for an off-shell thread $\int \Delta E(t)\, dt$ is an integral over a two dimensional surface the coordinates of which are the time t and a parameter U^0 measuring a distance along the thread. To obtain an integrand independent of the energy

E we let the range of U^0 be proportional to E. The form of the integrand of course depends upon the properties of the thread. But with appropriate properties of the thread our expression (11) could just be considered a penetration amplitude of this form.

A better approximation is to take functional integral over all possible ways that the system can pass the barrier. The integral (1) is to be considered an approximation of this type.

Another way to express the assumption that hadronic material is threads or sticks is the interpretation of the integrand of the Veneziano-type integrand (11) as the contribution from a class of very complex Feynman diagrams. More precisely speaking, it seems intuitively reasonable that a possible physical interpretation of the generalized Veneziano model could be that the surfaces mentioned in the foregoing section are rough pictures of very complicated Feynman diagrams,, that is to say we assume that only Feynman diagrams having the large scale topological structure of a two-dimensional network are of importance. Feynman diagrams namely have some multiplicative structure in them and so a complicated one can be expected to be of the form of the exponential of an integral over some parametrisation of the vertices and/or the propagators. Now, however, a Feynmann diagram is only a pure product before the integration over the loop momenta has been carries out and so we have to integrate the exponential over the loop momenta. If only a rather narrow set of loop momentum values contribute essentially to the integral we can do alone with a neighborhood of the point where the exponent takes on its maximum value.

Since fourmomentum (and also charge) is conserved at every vertex we can approximately describe a given set of fourmomenta through the propagators of a ~~x~~ very complex surface-like diagram by a set of fields $\vec{f}^{\mu}$ in the same way as a stationary flow ~~f~~ of some kinds of fluids. A choice of loop momenta over which to integrate then corresponds ~~then~~ to choosing an expansion of the form

$$\vec{f}^{\mu}(\underline{X}) = \sum_{\phi} \chi_{\phi} \, \vec{f}^{\mu}_{\phi}(\underline{X}) + \vec{f}^{\mu}_{0}(\underline{X})$$

where each of the fields $\vec{f}^{\mu}_{\phi}$ ~~xxx~~ is free of sources and sinks and independent of the positions of the external lines around the boundary of $\mathscr{S}$ and of the fourmomenta of the external mesons. The inhomogeneous term $\vec{f}^{\mu}_{0}$ on the other hand has the same sources (corresponding to the external mesons) as $\vec{f}^{\mu}$ itself.

Heuristic arguments can be given that the integrand of our type of Feynman diagrams has the momentum dependence

$$\exp\left(-\pi\alpha' \int_{\mathscr{S}} \vec{f}^{\mu} \cdot \vec{f}_{\mu} \, d\underline{X}\right)$$

(This is at least the simplest form compatible with Lorentz invariants)
The integration over loop momenta is now easily calculable

Feynman diagram $\approx$

$$\approx \int \exp\left(-\pi\alpha' \int_{\mathcal{G}} \vec{f}^{\mu}\cdot\vec{f}_{\mu}\, d\underline{x}\right) \prod_{\phi} dX_{\phi}$$

$$= \exp\left(-\pi\alpha' \text{ extremal value of} \int_{\mathcal{G}} \vec{f}^{\mu}\cdot\vec{f}_{\mu}\, d\underline{x}\right)$$

$$\cdot \int \exp\left(\sum_{\phi,\psi} c_{\phi\psi} X'^{\beta}_{\phi} X'_{\psi}\right) \prod_{\phi} dX'_{\phi}$$

where

$$c_{\phi\psi} = \int_{\mathcal{G}} \vec{f}^{\mu}_{\phi}\cdot\vec{f}_{\mu\psi}\, d\underline{x}$$

is independent of the inhomogeneous term and thus of the positions of the external tions and the external meson momenta. The new integration variables χ'_ϕ are just the deviations of χ_ϕ from the values of χ_ϕ giving the integral its extremal value . The integral

$$\int \exp\Big(\sum_{\psi,\phi} c_{\phi\psi}\, \chi'_\phi \chi'_\psi\Big) \prod_\phi d\chi'_\phi$$

is just an (in fact infinite) constant under changes of the positions of the external lines and the external momenta.

Treating the summation over diagrams with various charge states of the internal particles leads us analogously to that we obtain contributions of the form (11) from the considered class ~~through~~ of Feyman diagrams with the rough structure of the surface $\mathcal{S}$ and with the external lines in given positions .

The integration of equation (1) corresponds to the summation over classes of Feyman diagrams in which the external lines meet the diagram at different positions relative to the large scale structure.

We hope later to consider our model in more detail from point of view of complicated Feynman diagrams.

Section III.

Why should the structure of the most important Feynman diagrams just be twodimensional ?

This question is equivalent to the question: Why should hadronic material just have a threadlike structure and not for instance ~~fxxx~~ form of balls ?

We would like to argue that if it is assumed that infinitely complex Feynman diagrams having the rough structure of a piece of a D-dimensional manifold are dominant, the only nontrivial possibility is D=2.

Let us consider what happens when the same piece of manifold M of dimension D is filled with more and more complex Feynman diagrams. Especially compare two diagrams one of which has roughly l^D times as many vertices and propagators as the other one and a distance between neighboring vertices that is roughly l^{-1} times as long, both diagrams attached to the same D-dimensional piece. Now how does the parameters of the l^D times as dense diagram behave as a function of l ?

For a given flow fields $\vec{j}^{\mu}$ and $\vec{g}^{1}$ the flux through each propagator varies roughly like l^{-D+1}. This is so because the number of propagators to bring the flux through a hypersurface

(of dimension D-1) inside $\mathcal{M}$ varies as l^{D-1}. Because of Lorentz invariance the lowest power of the fourmomentum that can occur in power series expansion of a vertex or a propagator is the second one (apart from the zeroth) ; this one varies like $l^{-2(D-1)}$, and since the number of propagators or vertices varies like l^D the upon the fields depending factor has its logarithm proportional to $l^D \cdot l^{-2(D-1)} = l^{2-D}$. So the amplitude is an integral over an expression of the form

$$C \exp\left(G\, l^{2-D}\right)$$

where only G contains the kinematics and where the limit $l \to \infty$ has to be taken. That is to say that unless

$$D \leq 2$$

the amplitude will not at all depend upon kinematics i. e. it will be a constant.

The case $D=1$ is like $D \geq 3$ rather trivial; in fact it will describe non interacting bound states of the elementary particles. This might be relevant for leptons(allthough it might be more interesting also to consider those as threads) but for the description of hadron physics only the case $D=2$ is left.

After the completion of the main part of this work, we found that similar work had been done by Leonard Susskind.

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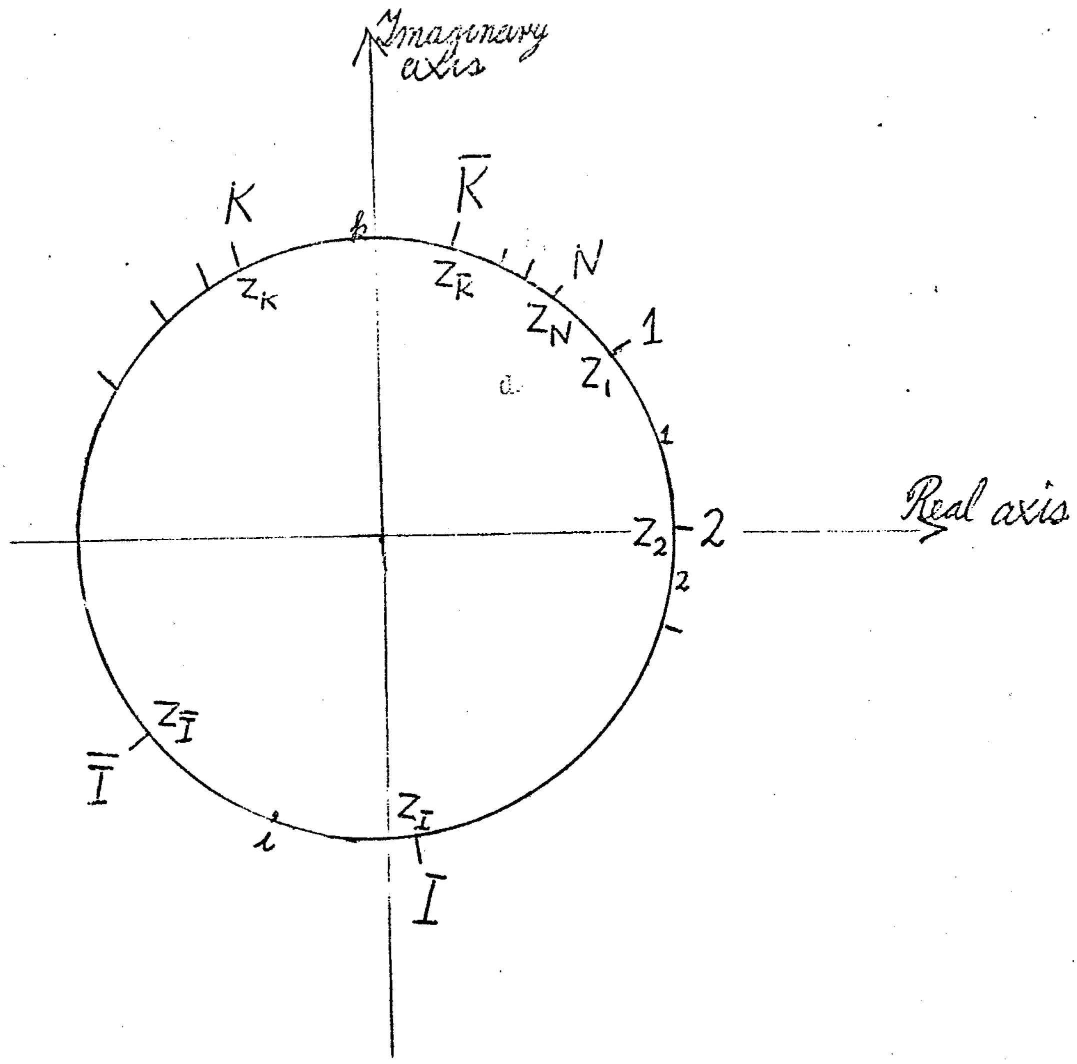

Fig 1.

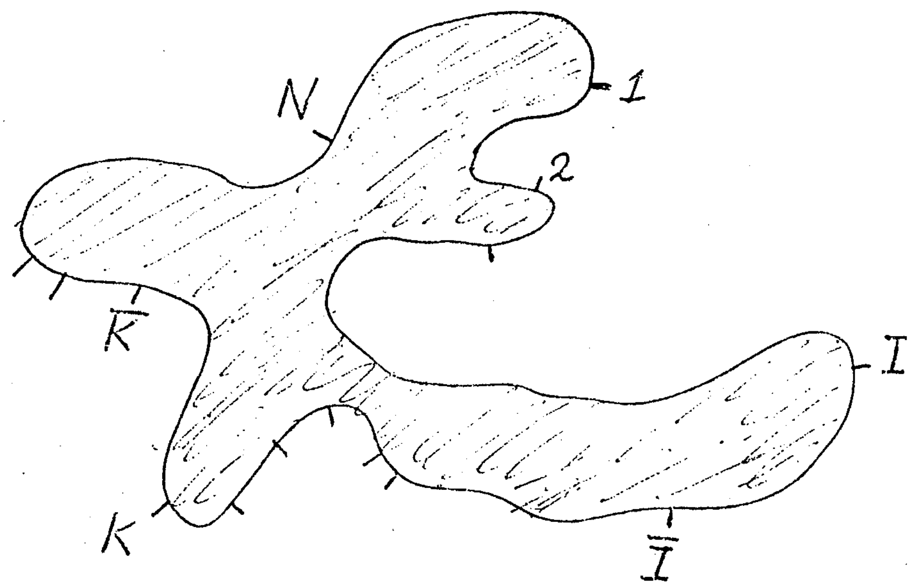

Fig 2.
A possible domain $\mathcal{S}$.

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